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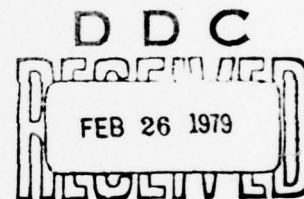
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CHANNEL BLOCKING IN A
SATELLITE COMMUNICATION SYSTEM MODEL

by

D. P. GAVER

and

J. P. LEHOCZKY

October 1978

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CHANNEL BLOCKING IN A SATELLITE COMMUNICATION SYSTEM MODEL^{*}

by

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1. Formulation of the Problem

Consider a communication system consisting of r stations, each of which must be able to communicate with all the others. The communication is conducted via an intermediate satellite. Since each station has, realistically, a finite capacity to handle messages simultaneously in progress, and since the satellite itself has limited capacity, the system will sometimes be congested, and a message applying for transmission will be blocked, i.e. effectively given a busy signal. We wish to calculate the probability that a message will be blocked, or delayed. The reader familiar with telephone system congestion theory, see Syski [1960], or Cooper [1972], will recognize this as a more complicated version of the situation for which the "Erlang B" formula--a truncated Poisson--holds. Essentially we assume that blocked calls are lost. For a model describing the more realistic "re-try" situation. see Gaver and Lehoczký [1976].

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Specifically, we assume that station i ($1 \leq i \leq r$) has c_i channels, and the satellite has c_s channels. We assume that a message initiated at station i and intended for station j (i to j for short) requires a free channel at i , one at the satellite, and one at j before transmission may begin. If no channel is available at one of these three locations, blockage occurs. We assume the satellite offers direct access, thus if any channel is available in the satellite a user will not be blocked at that point. Furthermore we assume that a channel in use is not available to any other user. That is, there is no possibility of simultaneous transmission by another user on an occupied channel and consequent message spoilage. The possibility of message destruction apparently exists for some existing satellite communication systems; see Kleinrock [1975], and Gaver and Lehoczky [1977]. Finally, direct access structure is apparently not yet available in practice, according to our information. Our study pertains to conceptual systems.

2. Analysis

Suppose that attempts to transmit messages from i to j arrive according to a homogeneous Poisson process with rate λ_{ij} , the rate of message termination, when the call is from i to j , is μ_{ij} , and holding times are independently exponential. Let $\rho_{ij} = \lambda_{ij}/\mu_{ij}$, and $\eta_{ij} = \rho_{ij} + \rho_{ji}$. Let $X_{ij}(t)$ be the number of messages or calls in progress from i to j at time t . It is clear from our formulation that $\underline{X}_t = \{X_{ij}(t), 1 \leq i < j \leq n\}$ is a multivariate Markov process in continuous time.

Steady-State Solution

Note that \underline{X}_t satisfies inequality constraints C which occur because the channel capacity is limited at the various stations.

$$C: X_{12}(t) + X_{13}(t) + \dots + X_{1r}(t) + X_{21}(t) + \dots + X_{r1}(t) \leq C_1,$$

and in general,

$$\sum_{j \neq i} X_{ij}(t) + \sum_{j \neq i} X_{ji}(t) \leq c_i, \quad 1 \leq i \leq n$$

$$\sum_{i=1}^r \sum_{j \neq i} X_{ij}(t) \leq c_s. \quad (2.1)$$

If the constraints were not present ($c_i = \infty, c_s = \infty$) then $X_{ij}(t)$ is an infinite server, Poisson arrival queueing process (termed M/M/ ∞) for every station pair i, j , and the stationary distribution is then Poisson:

$$\lim_{t \rightarrow \infty} P\{X_{ij}(t) = n_{ij}\} = e^{-\rho_{ij}} \frac{\rho_{ij}^{n_{ij}}}{n_{ij}!}, \quad n_{ij} = 0, 1, 2, \dots \quad (2.2)$$

Furthermore, the number of calls in progress between all pairs of stations are independent. It may even be stated that the above, (2.2), is true for the arbitrary service time situation. If the constraints are seldom binding, that is if blocking is a rare event, then (2.2) provides a useful approximation.

In the case that the constraints are imposed, the above result is also very nearly true, as is seen from the following:

Result. The stationary joint distribution of X_{ij} is Poisson constrained to the region C . That is,

$$\begin{aligned} \lim_{t \rightarrow \infty} P\{X_{ij}(t) = n_{ij}\} &\equiv \lim_{t \rightarrow \infty} P\{X_{12}(t) = n_{12}, \dots, X_{r,r-1}(t) = n_{r,r-1}\} \\ &= \pi_n = \prod_{i \neq j} e^{-\rho_{ij}} \frac{\rho_{ij}^{n_{ij}}}{n_{ij}!} / \sum_{\substack{n \\ n \in C}} \prod_{i \neq j} e^{-\rho_{ij}} \frac{\rho_{ij}^{n_{ij}}}{n_{ij}!} \end{aligned} \quad (2.3)$$

if $n_{ij} \in C$, and is zero otherwise. That is, if

$$n_{12} + n_{13} + \dots + n_{1r} + n_{21} + \dots + n_{r1} \leq c_1$$

etc., as in (2.1).

Discussion. In order to justify the solution (2.3) we consider the balance equations for the steady state probabilities π . These have the following form at state values away from the boundaries, the latter being defined by the constraint set C .

$$\begin{aligned} & \pi(n_{12}, \dots, n_{1r}, n_{21}, \dots, n_{2r}, \dots, n_{r1}, \dots, n_{r,r-1}) \\ & \times [\lambda_{12} + \dots + \lambda_{r,r-1} + \sum_{i \neq j} n_{ij} \mu_{ij}] \\ & = \sum_{i \neq j} \pi(\dots, n_{ij}+1, \dots) [(n_{ij}+1) \mu_{ij}] \\ & + \sum_{i \neq j} \pi(\dots, n_{ij}-1, \dots) \lambda_{ij} . \end{aligned} \quad (2.4)$$

These equations state that the rate of departure from the state \underline{n} equals the rate at which that state is entered. Actually, there is local balance: if $\pi(n_{ij})$ denotes marginal distribution of calls in progress between i and j , then in the unconstrained case we can see that local balance holds: if

$$\pi(\underline{n}) = \prod_{i \neq j} \pi(n_{ij}) \quad (2.5)$$

and

$$\pi(n_{ij}) = e^{-\rho_{ij}} \frac{\rho_{ij}^{n_{ij}}}{n_{ij}!} \quad (2.6)$$

Then, termwise in (2.4) for all $i \neq j$,

$$\begin{aligned} \pi(\dots, n_{ij}, \dots) [\lambda_{ij} + n_{ij} \mu_{ij}] \\ = \pi(\dots, n_{ij}+1, \dots) (n_{ij}+1) \mu_{ij} + \pi(\dots, n_{ij}-1, \dots) \lambda_{ij} \end{aligned} \quad (2.7)$$

since

$$e^{-\rho_{ij}} \frac{\rho_{ij}^{n_{ij}}}{n_{ij}!} \lambda_{ij} = e^{-\rho_{ij}} \frac{\rho_{ij}^{n_{ij}+1}}{(n_{ij}+1)!} (n_{ij}+1) \mu_{ij} \quad (2.8a)$$

$$e^{-\rho_{ij}} \frac{\rho_{ij}^{n_{ij}}}{n_{ij}!} n_{ij} \mu_{ij} = e^{-\rho_{ij}} \frac{\rho_{ij}^{n_{ij}-1}}{(n_{ij}-1)!} \lambda_{ij} \quad (2.8b)$$

This shows that the product from solution (2.5) holds for n strictly within C . Now suppose n is a boundary point. This means that some transition rates which were $\lambda_{ij} > 0$ in the unconstrained case must be equal to zero, in order to keep the X process within C , i.e. on the left hand side of the balance equations (2.4) these terms now involve zeros

for λ_{ij} . But examination of (2.7) shows that if $\lambda_{ij} = 0$ then adding one to n_{ij} results in $n_{ij}+1$ --a state outside C . Consequently, we define $\pi(\dots, n_{ij}+1, \dots) = 0$. But according to (2.8b), balance still holds. Consequently, the solution in the constrained case is just the product form (2.5), constrained to fall within C , as expressed by (2.3).

Example. Suppose that two stations communicate via satellite. The constraint set, C , is

$$\begin{aligned} n_{12} + n_{21} &\leq c_1 \\ n_{21} + n_{12} &\leq c_2 \\ n_{12} + n_{21} &\leq c_s . \end{aligned} \quad (2.9)$$

In this case the smallest channel capacity, be it at station 1, 2, or satellite, determines C . The balance equations are

$$\begin{aligned} &\pi(n_{12}, n_{21}) [\lambda_{12} + \lambda_{21} + n_{12}\mu_{12} + n_{21}\mu_{21}] \\ &= \pi(n_{12}+1, n_{21}) (n_{12}+1)\mu_{12} + \pi(n_{12}, n_{21}+1) (n_{21}+1)\mu_{21} \\ &\quad + \pi(n_{12}-1, n_{21}) \lambda_{12} + \pi(n_{12}, n_{21}-1) \lambda_{21} . \end{aligned} \quad (2.10)$$

Clearly, we define $\pi(n_{12}, n_{21}) = 0$ if $n_{12} + n_{21} > \min(c_1, c_2, c_s)$.

Now inside C the balance equations (2.10) are satisfied by the product form

$$\begin{aligned}\pi(n_{12}, n_{21}) &= \left(e^{-\rho_{12}} \frac{\rho_{12}^{n_{12}}}{n_{12}!} \right) \left(e^{-\rho_{21}} \frac{\rho_{21}^{n_{21}}}{n_{21}!} \right) \\ &= \pi(n_{12}) \pi(n_{21}) .\end{aligned}\quad (2.11)$$

Now suppose $n_{12} + n_{21} = \min(c_1, c_2, c_3)$, i.e. is on the boundary of C . Then $\lambda_{12} + \lambda_{21}$ must be set equal to zero. But, correspondingly $\pi(n_{12}+1, n_{21}) = \pi(n_{12}, n_{21}+1) = 0$. By local balance, the product form solution continues to hold on the boundary. Write for $n_{12} + n_{21} = \text{boundary point}$

$$\begin{aligned}&\left(e^{-\rho_{12}} \frac{\rho_{12}^{n_{12}}}{n_{12}!} \right) \left(e^{-\rho_{21}} \frac{\rho_{21}^{n_{21}}}{n_{21}!} \right) [0 + 0 + n_{12}\mu_{12} + n_{21}\mu_{21}] \\ &= 0 + 0 + \left(e^{-\rho_{12}} \frac{\rho_{12}^{n_{12}-1}}{(n_{12}-1)!} \right) \left(e^{-\rho_{21}} \frac{\rho_{21}^{n_{21}}}{n_{21}!} \right) \lambda_{12} \\ &\quad + \left(e^{-\rho_{12}} \frac{\rho_{12}^{n_{12}}}{n_{12}!} \right) \left(e^{-\rho_{21}} \frac{\rho_{21}^{n_{21}-1}}{(n_{21}-1)!} \right) \lambda_{21}\end{aligned}\quad (2.12)$$

and cancel off common factors; the balance is obvious. It is only necessary to normalize the product form over the constraint region, as dictated by (2.3).

3. Blocking Probabilities

The probability that a call originating at station i is blocked, essentially receiving a busy signal, is calculated in principle from (2.3). It is convenient to define

$Y_{ij}(t) = X_{ij}(t) + X_{ji}(t)$ for $1 \leq i, j \leq r, i \neq j$ and $Y_{ii}(t) = 0$. Here $Y_{ij}(t)$ represents the total number of calls in progress between stations i and j . In steady state Y_{ij} are independent Poisson random variables with parameter $\eta_{ij} = \rho_{ij} + \rho_{ji}$, constrained by C:

$$\sum_{j=1}^r Y_{ij} \leq c_i, \quad 1 \leq i \leq r \quad (3.1)$$

$$\frac{1}{2} \sum_i \sum_j Y_{ij} \leq c_s.$$

Now observe that a call from i to j can be blocked in three ways:

- 1) At the originating station, if Station i is full.

This is event $E_i = \{\sum_{j=1}^r Y_{ij} = c_i\}$.

- 2) If the satellite channels are full, the event

$E_s = \{\sum_i \sum_j Y_{ij} = 2c_s\}$.

- 3) If the destination station, Station j , is full. This is event E_j .

The probability an i to j or j to i transmission is blocked somewhere is given by

$P(i \text{ to } j \text{ or } j \text{ to } i \text{ transmission is blocked})$

$$\begin{aligned}
 &= P(E_i \cup E_j \cup E_s) \\
 &= P(E_i) + P(E_j) + P(E_s) - P(E_i \cap E_j) \\
 &\quad - P(E_i \cap E_s) - P(E_j \cap E_s) + P(E_i \cap E_j \cap E_s) . \quad (3.2)
 \end{aligned}$$

Each of the above probabilities can be represented in terms of the Y_{ij} random variables. The value of each of these probabilities can be easily found by summing terms of the form (2.3), the steady state distribution, over a boundary portion of C . For example

$$P(E_i) = \sum_{\substack{Y \in C \\ \sum_{j \neq i} Y_{ij} = c_i}} \prod_{k < \ell} \frac{Y_{k\ell} \eta_{k\ell}}{Y_{k\ell}!} \bigg/ \sum_{Y \in C} \prod_{k < \ell} \frac{Y_{k\ell} \eta_{k\ell}}{Y_{k\ell}!} , \quad (3.3)$$

while other terms in (3.2) can be computed by changing the numerator to reflect a change in the boundary conditions.

It is clear that the calculation of each of the terms in (3.2) is in principle straightforward as it involves merely the calculation of a well-defined ratio. Unfortunately, the problem may be nearly computationally infeasible if the c_i 's, c_s , and k are large. For example if $c_i = c_s$, $1 \leq i \leq r$, then C includes

$$\binom{c_s + \binom{k}{2}}{\binom{k}{2}}$$

distinct points. If $c_s = 50$, then for $k = 3, 4$, and 5 this quantity is 2.3426×10^4 , 3.2468×10^7 , and 7.5394×10^{10} respectively. Many interesting cases are essentially computationally infeasible.

Computer programs have been written for the cases of $r = 3$ and 4 ($r = 2$ can be done with the Erlang B formula). It is possible to reduce the computations necessary in (3.3) as follows. Let $c_{\min} = \min(c_1, \dots, c_r, c_s)$. The denominator (and numerator) can be rewritten as

$$\begin{aligned} \sum_{n=0}^{c_s} \sum_{\substack{y \in C \\ \sum_{k < \ell} y_{k\ell} = n}} \prod_{k < \ell} \frac{\eta_{k\ell}^{y_{k\ell}}}{y_{k\ell}!} &= \sum_{n=1}^{c_s} S_n \\ &= \sum_{n=0}^{c_{\min}} S_n + \sum_{n=c_{\min}+1}^{c_s} S_n. \end{aligned} \quad (3.4)$$

Now using the multinomial theorem

$$\begin{aligned} \sum_{n=0}^{c_{\min}} S_n &= \sum_{n=0}^{c_{\min}} \sum_{\substack{y \in C \\ \sum_{k < \ell} y_{k\ell} = n}} \prod_{k < \ell} \frac{\eta_{k\ell}^{y_{k\ell}}}{y_{k\ell}!} = \sum_{n=0}^{c_{\min}} \sum_{\substack{y \in C \\ \sum_{k < \ell} y_{k\ell} = n}} \prod_{k < \ell} \frac{\eta_{k\ell}^{y_{k\ell}}}{y_{k\ell}!} \\ &= \sum_{n=0}^{c_{\min}} \frac{(\sum_{k < \ell} \eta_{k\ell})^n}{n!}. \end{aligned} \quad (3.5)$$

The last term is simply computed. This observation removes

$$\binom{c_{\min} + \binom{k}{2}}{\binom{k}{2}}$$

can reduce the computations required substantially. Nevertheless, for interesting values of k , c_s , and c_i , $1 \leq i \leq r$, the number of terms needed to be computed may render the method to be infeasible. Research directed toward finding a tractable approximation useful for large networks is presently under way.

4. Numerical Results for Three and Four Stations

We now present a few numerical results that have been obtained for the situation in which three or four ground stations communicate via satellite. The computer programs used for obtaining these numbers is available upon request. It calculates the probabilities using enumeration of the multinomial terms. Three stations require a relatively small number of computations. For the case of four stations, the reduction (3.5) is utilized.

We are interested in cases where the blocking probabilities are small, say less than .10. We wish to see if in such circumstances probability of blocking $(P(E_i \cup E_j \cup E_s))$ can be estimated assuming independence. Specifically, we wish to determine if $P(E_i \cup E_j \cup E_s)$ can be approximated by $1 - P(\bar{E}_i) P(\bar{E}_j) P(\bar{E}_s)$. If such an approximation is reasonable, it reduces the amount of computation required in the problem. In looking over the following tables, it appears that this approximation is usefully accurate, especially for the cases of small (less than .1) block probability.

Probability a 1 to 2 or 2 to 1 Message is Blocked

Given System Specifications

Case 1. $r = 3, c_1 = c_2 = c_3 = 10, c_s = 12, \eta_{12} = \eta_{13} = \eta_{23} = \eta$

η	Exact $(P(E_1 \cup E_2 \cup E_s))$	Approximate $(1 - P(\bar{E}_1)P(\bar{E}_2)P(\bar{E}_s))$
2.0	.014	.017
3.0	.090	.108
4.0	.206	.241
5.0	.316	.361

Case 2. $r = 4, c_1 = c_2 = c_3 = 10, \eta_{ij} = 1.0$

c_s	Exact	Approximate
10	.0431	.0432
12	.0119	.0155
14	.0030	.0033
16	.0017	.0017
18	.0016	.0016
20	.0016	.0016

Case 3. $r = 4, c_1 = c_2 = c_3 = 10, \eta_{ij} = 2.0$

<u>c_s</u>	<u>Exact</u>	<u>Approximate</u>
10	.3019	.3023
12	.1991	.2038
14	.1226	.1335
16	.0805	.0916
18	.0683	.0733
20	.0674	.0692

Case 4. $r = 4, c_1 = c_2 = c_3 = 10, \eta_{ij} = 3.0$

<u>c_s</u>	<u>Exact</u>	<u>Approximate</u>
10	.493	.494
12	.402	.408
14	.319	.339
16	.254	.286
18	.221	.242
20	.216	.223

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